1901001101060001
EXAMINATION FEBRUARY-MARCH 2024
MASTER OF ARTS PART-I
MATHEMATICS
P.D.E. & FOURIER ANALYSIS - LEVEL 6

[**Time: As Per Schedule]** **[Max. Marks: 100]**

Instructions:
1. Fill up strictly the following details on your answer book
a. Name of the Examination: **MASTER OF ARTS PART-1**
b. Name of the Subject: **MATHEMATICS P.D.E. & FOURIER ANALYSIS – LEVEL 6**
c. Subject Code No: **1901001101060001**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. There are five questions in this question paper.

Seat No:

Student's Signature

Q.1

A. Obtain the direction ratio of tangent line to the curve in space. **7**

B. Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1 \; and \; x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$. **7**

C. Find the orthogonal trajectories on the sphere $x^2 + y^2 + z^2 = a^2$ of its intersections with the paraboloids $xy = cz$, being a parameter. **6**

OR

A. Prove that paraffin differential equation always possesses an integrating factor. **7**

B. Find the integral curves of the equations $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^2-x^3)} = \frac{dz}{z^2(x^3-y^3)}$. **7**

C. In usual notation discuss the method for solving the equation $Pp + Qq = R$, where P, Q, R are functions of x, y and z. **6**

Q.2	A.	State and prove the necessary and sufficient condition for the pfaffian differential equation to be integrable.	**7**
	B.	Derive the condition for the partial differential equation $f(x,y,z,p,q) = 0$ and $g(x,y,z,p,q) = 0$ to be compatible.	**7**
	C.	Explain Natani's method to solve the pfaffian differential equation.	**6**

OR

	A.	Explain the method to obtain the orthogonal surfaces to the system of surface $f(x,y,z) = c$	**7**
	B.	Explain Charpit's method to solve the partial differential equation $f(x,y,z,p,q) = 0$	**7**
	C.	Solve the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ if it is integrable.	**6**
Q.3	A.	Show that the equations $xp - yq = x, x^2 p + q = xz$ are compatible and find their solution.	**7**
	B.	Prove that if$(\alpha_r D + \beta_{rD'} + \gamma_r)$is a factor of F(D,D') and d_r (ξ) is an arbitrary function of the single variable ξ then if $\alpha_r \neq 0, u_r = \exp\left(-\frac{\gamma_r x}{\alpha_r}\right)\phi(\beta_r x - \alpha_r y)$is a solution of the equation $F(D,D')z = 0$.	**7**
	C.	Show that the equation $Z = px + qy$ is compatible to any equation $f(x,y,z,p,q) = 0$ that is homogeneous in x, y and z and also solve the simultaneous equations, $z = px + qy$ and $2xy(p^2 + q^2) = z(yp + xq)$.	**6**

OR

	A.	Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy$.	**7**
	B.	Find the general solution of the partial differential equation $r + s - 2t = e^{x+y}$.	**7**
	C.	Prove that the equations $f(x,y,p,q) = 0$ and $g(x,y,p,q) = 0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,p)}{\partial(y,q)} = 0$.	**6**

Q.4 A. Find the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$ and prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots \dots = \frac{\pi^2}{12}$. **7**

B. Derive Fourier series formula for the interval $[0,2\pi]$. **7**

C. Prove that the sum of the squares of the Fourier co-efficients of a square integrable function always converges. **6**

OR

A. Expand the function $f(x) = x - x^2, -1 < x < 1$ in terms of Fourier series. **7**

B. Prove that every orthogonal system of functions is linearly independent system. **7**

C. The expansion co-efficient of L^2-integrable function converges to zero as n is increased indefinitely for an orthogonal system. **6**

Q.5 A. The temperature in semi-infinite rod which is bounded between $0 \le x < \infty$ is denoted by 'u' and governed by the equation $u_t = k\, u_{xx}$ subject to the conditions $u = 0\ at\ t = 0; x \ge 0, \frac{du}{dx} = -\mu;$ where μ is constant when $x = 0\ and\ t > 0.$
$u(x,t)$ is bounded then show that $u(x,t) = \frac{2}{\pi}\, \mu \int_0^\infty \frac{\cos px}{p^2}\left(1 - e^{-p^2 kt}\right)dp.$ **7**

B. Find the Fourier transform of the function $f(x) = \begin{cases} \frac{\sqrt{2\pi}}{2\varepsilon}; & |x| \le \varepsilon \\ 0; & |x| > \varepsilon \end{cases}$. **7**

C. Find the Fourier transform of the function $f(x) = \begin{cases} e^{iwx}; & a < x < b \\ 0; & x < a, x > b \end{cases}$. **6**

OR

A. Solve the Heat equation $u_t = u_{xx}$ subject to the following condition $u_x(0,t) = 0, u(x,0) = \begin{cases} x; & 0 \le x < 1 \\ 0; & x \ge 1 \end{cases}$ and $u(x,t)$ is bounded **7**

B. Find the Fourier transform of the function $f(t) = \begin{cases} 1 - \frac{|t|}{a}; & |t| < a \\ 0; & otherwise \end{cases}$ **7**

C. Find the Fourier transform of the function $f(x) = \begin{cases} 1; & |x| < 1 \\ 0; & |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty (\sin t)/t \ dt$ **6**

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